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The eccentric position of the heart in the mammalian body and optimal energy transfer in single tube models

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Abstract

Many phenomena cannot be explained by traditional haemodynamics models. For example, the hearts of all mammals are neither at one end of the circulatory system nor at the geometric centre. Based on a new circulation model, we report that if the heart is located at either of these two positions, the energy saving rule will be violated. We assume that the main arterial system is under a steady, distributed transverse vibration with the heart as the input power source. The equation of motion of the artery is governed by a new pressure wave equation with total energy. We analyse the effects of the heart position on the pressure pulse shape and the spectrum. By a simplifying T-tube model, we find that there are many harmonic oscillating modes for the overall arterial system. The position of the heart affects the weights of different modes. If the heart is at the midpoint or at one end of the body, none of the even harmonic modes can be excited. If the heart is at a third along the whole system, the third oscillation mode in the system is missing. Thus, from an efficiency point of view, this model gives a strong reason for all mammals' hearts being at an eccentric position. Tube simulations were carried out to confirm the theoretical prediction. A new standing wave model to analyse the variation of the pressure pulse shape along the artery is discussed. The interesting result indicates that our new pressure wave equation possesses a high problem solving potential. It provides a new tool for studying arterial dynamics.

Keywords: efficiency, circulation model, heart position

Introduction

The heart occupies an eccentric position in the mammalian body (O'Rourke and Avolio 1980), which is neither at the geometric centre nor at one end of the body. This phenomenon has not been explained by any current theory of haemodynamics.

Many problems in the arterial systems cannot be solved by either the long existing distributed school or the Windkessel school. Quick *et al* (1998) has pointed out two crucial steps to solving the problem. They are first to apply Fourier analysis to the experimental data, and second to abandon the assumption of infinite length of the artery.

In order to improve modern arterial dynamics we suggest another important step: using a new governing pressure equation with all important energies taken into account.

Most governing equations for the blood wave or the pressure wave in the circulatory system can be traced back to the Navier–Stocks equation for fluids (Milnor 1989, Nichols and O'Rourke 1998). All models concentrated on the blood flow in the axial direction of the artery.

Recently, we pointed out that, although the ultimate purpose of circulation is to transport blood all the way to the microcirculatory system, in the main arterial system, its main purpose is to transport energy. Just like an electric power generation system, the ultimate purpose is to deliver electricity to users. However, during the transportation, raising the voltage reduces the power loss. From this point of view, and as pointed out by Milnor (1989), the axial flow kinetic power is just a small portion of the ventricular output, and we (Lin Wang *et al* 2002) deduced that most of the energy transportation in the artery is via the transverse elastic vibration of the blood vessel and the adherent blood.

We (Lin Wang *et al* 2004a) have also derived a blood pressure wave propagation equation with all the important energies taken into consideration. We call it the pressure wave equation with total energy.

A new equation can be acceptable as a better model only if it can be used to study more problems and explain more phenomena in haemodynamics.

Based on the pressure wave equation with total energy, we (Lin Wang *et al* 2004b) proposed a 'frequency matching' rule and gave some conclusions about the efficiency of the arterial system. We were able to explain quantitatively how the heart rate relates to the mammal's dimensions. Avolio and Kerkhof (2004) commented that many of the concepts need further investigation and *in vivo* experimental validation.

We further used the equation to show how the physical conditions of different organs are reflected specifically in the pressure pulse spectrum of the peripheral artery (Jan *et al* 2003).

In this paper, we will apply the same equation to a simplified T-tube model (O'Rourke and Taylor 1967) to illustrate quantitatively how the heart position affects the pressure pulse and why the heart occupies an eccentric position in the mammalian body.

Theory

We assumed that the arterial system is in distributed transverse vibration motion governed by the pressure wave equation with total energy (Lin Wang *et al* 1997, Lin Wang *et al* 2004a)

$$\frac{\partial^2 P}{\partial t^2} + b\frac{\partial P}{\partial t} + \omega_0^2 P = V_\infty^2 \frac{\partial^2 P}{\partial z^2} + \frac{2\pi r_0}{L_m C_A} F_{\mathrm{ext}}. \tag{1}$$

The pressure variation $P(z, t)$ at axial position z and time t is defined as the difference between the internal fluid pressure $P_i(z, t)$ and the local static pressure $P_o(z)$. That is,

$$P(z, t) = P_i(z, t) - P_o(z). \tag{2}$$

Here C_A is the area compliance of the main artery, and L_m is the mass per unit axial length of the arterial vessel and the adherent fluid that are oscillating transversely together. Parameter b is a damping constant, r_0 is the static radius of the vessel, and ω_0 is the residual intrinsic angular frequency of the main artery. V_∞ is the high frequency phase velocity, and F_{ext} is the external force per unit length at position z.

This wave equation is applicable for the whole arterial system as long as the arterial tubes remain cylindrically symmetric. The parameters may vary by position. The solution of the equation for the whole system must meet some required boundary conditions whenever there are heterogeneities, bifurcations or side branch loadings.

Here we simplify the arterial system as an elastic tube of length L with uniform high frequency velocity V_∞ and assume that the heart generates a force source at position $z = \xi$. This is similar to the T-tube model (O'Rourke and Taylor 1967). At the two ends of the arterial system, we assume that the internal fluid pressures $P_i(0, t)$ and $P_i(L, t)$ maintain their static values $P_o(0)$ and $P_o(L)$ at all times t so that the boundary conditions for the pressure variation P becomes

$$P(z = 0, t) = 0 \qquad P(z = \mathrm{L}, t) = 0. \tag{3}$$

The heart ejects blood with period T and duration ΔT and contributes a periodic force pulse at $z = \xi$, which is near the position of the aortic arch. After a transient time, the whole circulation system is then in a distributed oscillatory state.

We may write the external force as

$$\begin{aligned} F_{\text{ext}}(z, t) &= F(t)\delta(z - \xi) \\ F(t) &= F_o \qquad NT - \tfrac{1}{2}\Delta T \leqslant t \leqslant NT + \tfrac{1}{2}\Delta T \\ &= 0 \qquad \text{otherwise,} \end{aligned} \tag{4}$$

where $N = 0, \pm 1, \pm 2, \ldots$.

The periodic pulse force can be expressed as a Fourier cosine series:

$$F(t) = \sum_{m=0}^{\infty} a_m \cos \omega_m t, \tag{5}$$

where

$$\omega_m = \left(\frac{2\pi}{T}\right) m \tag{6}$$

$$a_o = F_o \frac{2\Delta T}{T} \tag{7}$$

$$a_m = F_o \frac{2 \sin\left(m\pi \frac{\Delta T}{T}\right)}{m\pi}. \tag{8}$$

The governing equation is linear if the parameters in equation (1) are not pressure dependent. We may first consider only the specific response of the pressure with respect to the mth input harmonic force of angular frequency ω_m. Then we sum over m of the various ω_m to get the overall effect of the periodic heart pulse. Alternatively, mathematically, we first seek a solution of the partial differential equation

$$\frac{\partial^2 P_\omega}{\partial t^2} + b\frac{\partial P_\omega}{\partial t} + \omega_0^2 P_\omega = V_\infty^2 \frac{\partial^2 P_\omega}{\partial z^2} + F\delta(z - \xi)\, \mathrm{e}^{-\mathrm{i}\omega t} \tag{9}$$

that fits the boundary conditions in equation (3). Here $F = 2\pi r_0 a_m/(L_m C_A)$ and $\omega = \omega_m$.

By the Green's function method (Migulin 1983, Butkov 1968), we find that

$$P_\omega(z,t) = \sum_{n=1}^{\infty} A_n \sin\frac{n\pi\xi}{L}\sin\frac{n\pi z}{L}\,\mathrm{e}^{-\mathrm{i}(\omega t+\Phi_n)}, \tag{10}$$

where

$$A_n = \frac{2F}{\left[\left(v_n^2+\omega_o^2-\omega^2\right)^2 + b^2\omega^2\right]^{\frac{1}{4}}} \qquad \phi_n = \tan^{-1}\frac{-b\omega}{v_n^2+\omega_o^2-\omega^2}$$

with

$$v_n = \frac{n\pi V_\infty}{L} \qquad n = 1, 2, 3, \ldots.$$

The pressure response is composed of various harmonic modes with different amplitudes A_n. The amplitude maximum occurs at $\mathrm{d}A_n/\mathrm{d}\omega = 0$ under the condition that the angular frequency ω of the external force is equal to one of the natural angular frequencies ω_{Rn}, where

$$\omega_{Rn}^2 = v_n^2 + v_o^2 - \tfrac{1}{2}b^2 \qquad n = 1, 2, 3, \ldots. \tag{11}$$

If $v_o^2 - \frac{1}{2}b^2 = 0$ or is very small, then all the natural angular frequencies are integer multiples of the fundamental natural angular frequency v_1, where

$$v_1 = \frac{\pi V_\infty}{L} \tag{12}$$

$$\omega_{Rn} = n v_1. \tag{13}$$

In this model, the fundamental mode for the pressure has no node between the two ends, the second mode has a node at $z = L/2$, and the third mode has two nodes.

An impulse external force of a short duration ΔT is a special case of the periodic force given by equation (4) with the period T approaching infinity. By substituting $T \to \infty$ into equations (6), (7) and (8), we can see that the impulse force is composed of continuous angular frequencies of equal weights. Therefore the natural angular frequencies of the arterial system can be found by analysing its pressure impulse response in the frequency domain.

The effect of the heart position ξ on different harmonic modes appears through the weight function $W_n = \sin(n\pi\xi/L)$ in equation (10). If the heart is at the middle of the system, that is, if $\xi = L/2$, then $W_n = 0$ for all even n and it will not have a pressure response with even harmonics. If $\xi = L/3$, then $W_n = 0$ for $n = 3, 6, 9\ldots.$ If $\xi = L/4$, then $W_n = 0$ for $n = 4, 8\ldots.$ In order to retain all the possible lowest modes, the heart has to be at an eccentric position ξ so that $\sin(n\pi\xi/L)$ will not be zero.

For the special case that the heart is at one end of the arterial system, the solution becomes

$$P_\omega(z,t) = \sum_n A_n \cos\frac{n\pi z}{2L}\,\mathrm{e}^{-\mathrm{i}(\omega t+\Phi_n)} \qquad n = 1, 3, 5\ldots. \tag{14}$$

Thus only odd modes will be excited.

A tube simulation experiment was set up to verify the analytical solutions in equations (10) and (14).

Results of the simulation experiment

A tube simulation experiment was set up as in figure 1 (Lin Wang *et al* 2000). An elastic tube made of latex of length L was used to simulate the main arterial system. It was filled with water and was maintained at a length of 15% greater than its natural length. The ends were

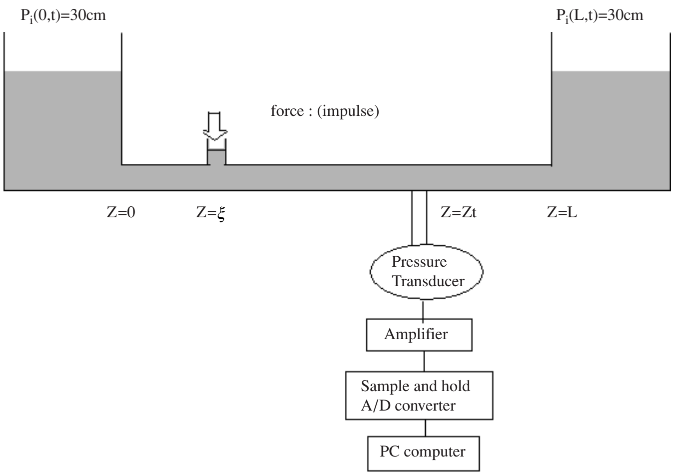
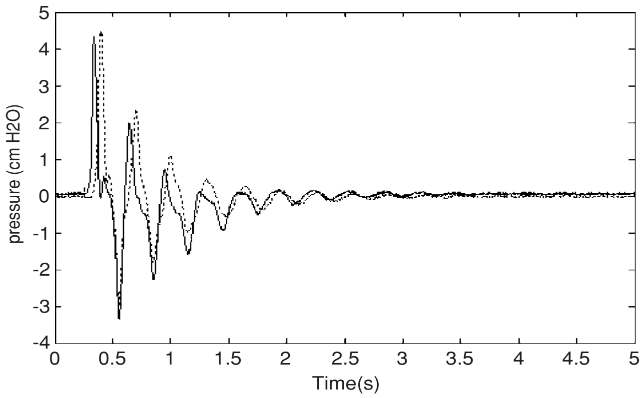
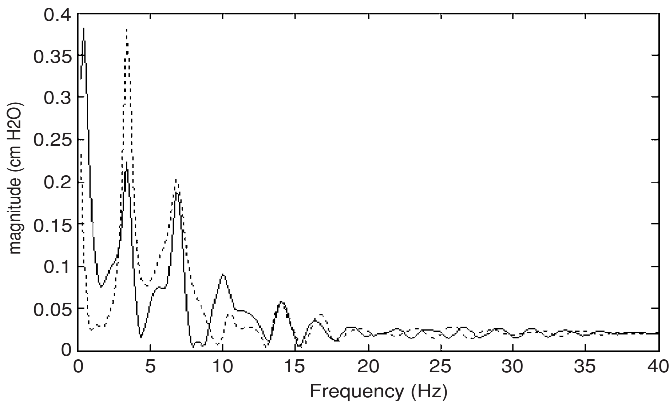


Figure 1. Experimental setup.

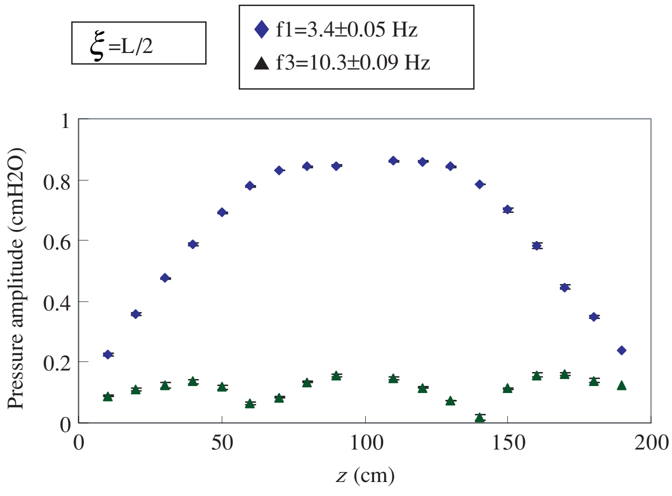


(a)

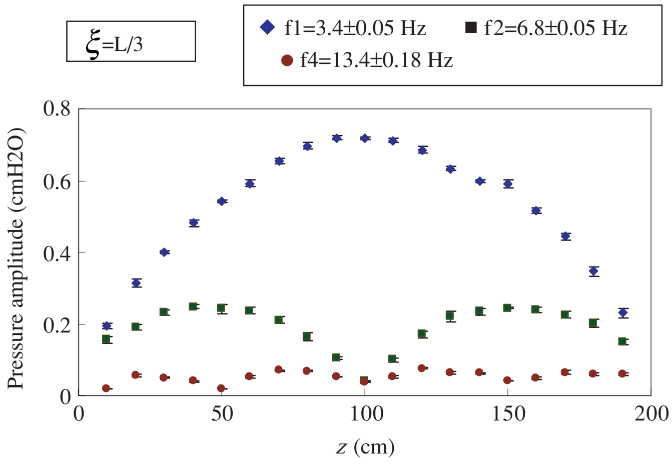


(b)

Figure 2. (a) Pressure impulse responses in the time domain for an $L = 200$ cm long tube with impulse force applied at $\xi = 22.5$ cm. Dotted line: measured at $z = 60$ cm; solid line: measured at $z = 40$ cm. (b) Same pressure impulse responses in the frequency domain.



(a)



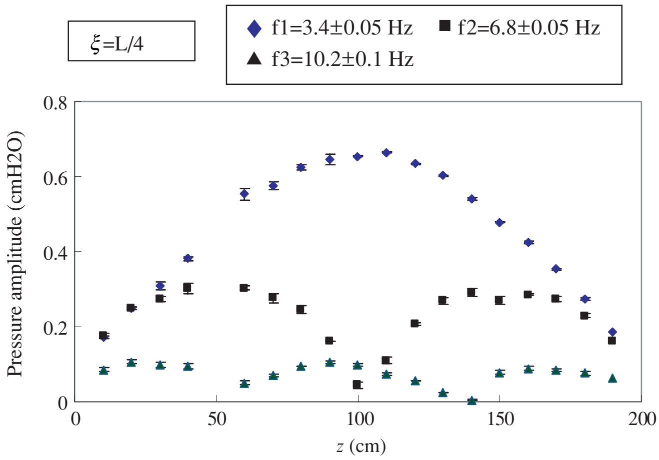
(b)

Figure 3. Amplitudes of the measured pressure impulse response at different z of an $L = 200$ cm long tube with impulses applied at (a) $\xi = L/2$, (b) $\xi = L/3$, (c) $\xi = L/4$ and (d) $\xi = L/8$. ◆: first resonant model; ■: second resonant mode; ▲: third resonant mode; ●: fourth resonant mode. I represents the SD (standard deviation) for five measurements on one tube.

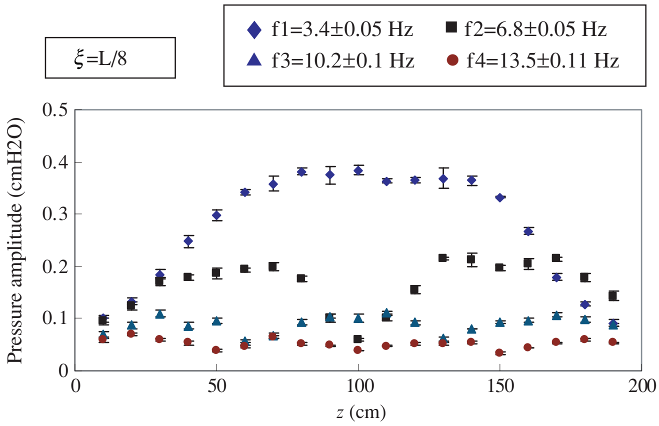
connected to two water containers with water levels 30 cm above the tube to simulate the constant pressures at the ends of the arterial system.

An EM controlled hammer produced 0.065 J of kinetic energy on a syringe with 2.0 cm^3 of water. The water ejection impulse was connected at $z = \xi$ to simulate the external impulse force generated by a heart. The pressure impulse responses were measured at various positions along the elastic tube by a DP103 differential pressure transducer (Validyne, USA). The time domain responses were digitized by an A/D converter with a sampling rate of 600 Hz, followed by a Fourier transform to get the frequency responses.

Figure 2 shows the measured pressure responses in the time and frequency domains for an elastic tube made of latex of length $L = 200$ cm, inner diameter $D_{\text{in}} = 9.50$ mm, outer diameter $D_o = 12.7$ mm, and tube thickness $h = 1.59$ mm. The position of the applied impulse



(c)



(d)

Figure 3. (Continued.)

is at $\xi = L/8$. The pressure variation P measured at $z_1 = 40$ cm is depicted as a solid line, and measured at $z_2 = 60$ cm as a dotted line.

Figure 3 shows the amplitudes of the first, second, third and fourth harmonics of the pressure variation recorded at 10-cm intervals along the elastic tube for input forces at various positions ξ: (3a) for $\xi = L/2$, (3b) for $\xi = L/3$, (3c) for $\xi = L/4$ and (3d) for $\xi = L/8$. The measured results confirm the theoretical predictions that for ξ at the midpoint of the tube, no even modes can be excited; for $\xi = L/3$, the third mode is absent; for $\xi = L/4$, the fourth mode is gone. For all these input positions, five measurements were performed on one tube and the fundamental natural frequency was $f_1 = 3.4 \pm 0.05$ Hz.

Figure 4 illustrates the same recording for an input force at one end of the tube. In this case, the fundamental natural frequency changes to $f_1 = 1.8 \pm 0.05$ Hz, and only modes of odd integer multiples of this frequency exist on the tube as predicted by equation (14).

Discussion

In the time domain, the heart generates a periodic force to the arterial system. In the frequency domain, this force is composed of all integer harmonic forces of modes $n = 1, 2, 3 \ldots$.

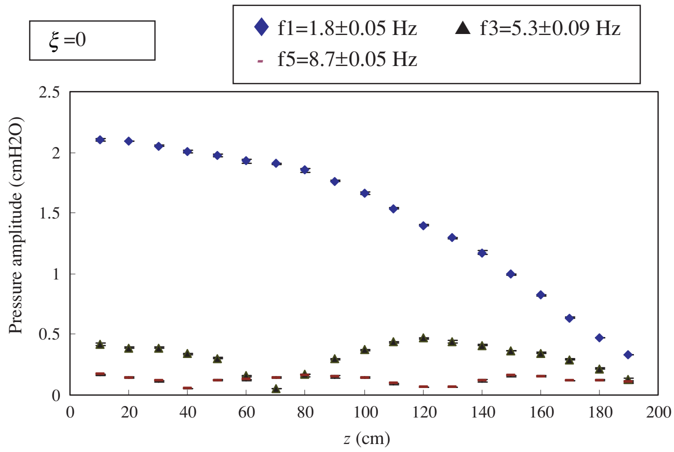


Figure 4. Amplitude of the measured pressure impulse response at different Z of an $L = 200$ cm long tube with impulse applied at one end of the tube. ♦: first resonant mode; ■: second resonant mode; ▲: third resonant mode; ●: fourth resonant mode; –: fifth resonant mode. I represents the SD (standard deviation) for five measurements on one tube.

An efficient arterial system should respond with all the harmonic modes so that no energy generated by the heart is wasted.

The response of the arterial system depends on the position ξ of the heart. If the heart were at the midpoint of the artery, figure 3(a) shows that the second mode does not exist along the whole tube. This is also consistent with the theoretical prediction. When $\xi = L/2$, through the factor $W_n = \sin(n\pi\xi/L)$ in equation (10), all even modes are missing.

If the heart were at one end of the artery, as figure 4 illustrates, only odd modes would be excited; this is also in agreement with the theoretical prediction in equation (14).

These results explain why the heart is neither at the centre nor at one end of the systemic circulatory system for mammals. In these positions, all the even modes of force generated by the heart would be wasted and this would be in contradiction of energy saving.

The weights of different modes also depend on the position through the factor $\sin(n\pi z/L)$; therefore the overall waveform varies from place to place. Since the first mode has no node for the pressure variance P, there is no absolute node along the whole tube in our simulation experiments. The shape of pressure P varies along the tube; figure 2(a) is an example. The peak of P at some point ($z = 60$ cm) farther away from the force source ($\xi = 22.5$ cm) may be even higher than the point ($z = 40$ cm) nearer to the force source. The result provides another explanation why mammal's pulsatile pressure increases as it travels away from the heart (Milnor 1989).

The $\sin(n\pi z/L)$ factor in equation (10) also predicts a standing wave behaviour for different modes. Figure 3 shows the amplitudes versus position for the first few harmonic modes of the pressure for impulses applied at different ξ. The existence of a standing wave does not need a complete occlusion or a complete opening at the two ends of the tube; it requires only that the pressures at the ends of the artery maintain their static values.

In equation (3), we assumed that the pressures maintained their static values at the two ends. These boundary conditions are not essential for the energy saving discussion. If the two ends remain as antinodes for the pressure wave, we can still reach similar conclusions about the position of the heart. Determining real end conditions for different species requires more *in vivo* studies.

Milnor (1989) has stated that many findings support the conclusion that the arterial system is approximately linear with respect to impedance. In our experimental tube simulation, due to the small input force we have applied, the pressure oscillations are small compared to the much larger oscillations in arteries. Equation (1), the pressure wave equation with total energy, is a linear differential equation as long as the coefficients are not pressure dependent in the physiological range. Since the solution of a linear equation is linearly proportional to the input force, we predict a quite similar result in the real arterial system with magnification in the pressure oscillations only.

The uniform T-tube model used in the simulation experiment seems quite different from a physiological artery. The actual arterial system has bifurcations, tapers, changes in calibre and elastic non-uniformity. In addition, it is connected to microcirculatory loads.

In our previous paper (Lin Wang *et al* 2004a), we have discussed that a geometric tapering or bifurcation does not necessarily induce reflection if the tube adjusts its elastic properties and mass per unit length so that there is no mismatch in pulse wave velocity. That is, a uniform tube requires that the pulse wave velocity be uniform along the tube only. The condition of the tube with non-uniform velocity can be solved by a length scaling with respect to the velocity or the wavelength first. A similar consideration still applies to the oscillating modes.

As a first order approximation, we may take the main artery to be a uniform T-tube and consider the effects of the microcirculatory loads as a perturbation. For example, we (Jan *et al* 2003) have treated the organ as a secondary small heart that generates harmonic forces with maximum amplitude near the organ's own natural frequency. Due to the linearity of the governing equation, as a first order approximation, the effects of the organs on the pressure will superpose on the final solution and will not affect the contribution from the heart.

From the above result, it seems that our new pressure wave equation possesses a high problem solving potential. It provides a new tool for studying modern arterial dynamics.

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